The Need for Differentiating Mathematics Instruction

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Of all the challenges facing educators today, perhaps, the most difficult is meeting the needs of individual learners. Within any given classroom, there will be a heterogeneous mix of ability levels—from students who perform above grade level to those who struggle to meet grade-level expectations. Yet, while differentiating reading instruction has become the norm in most classrooms, when it comes to teaching mathematics, targeting learning to the needs of individual students is much less common. In order to truly prepare students for success in and out of the classroom, teachers must differentiate the mathematics curriculum to meet the needs of all learners—by remediating or accelerating instruction, when necessary, and providing them with opportunities to learn and grow.

Differentiating Instruction with The Quantile Framework for Mathematics

ASCD (Association for Supervision and Curriculum Development) (2009) describes differentiated instruction as a means of creating multiple paths so that students of varying abilities, interests and learning needs experience equally appropriate ways to absorb, use, develop and present concepts as a part of the daily learning process. In reading, MetaMetrics®, Inc.'s widely adopted Lexile Framework® for Reading measures both reading ability and text difficulty on a single Lexile® scale, enabling teachers to match students with materials that meet and challenge their abilities. Similarly, The Quantile Framework® for Mathematics measures student mathematical ability, the curriculum and teaching materials on the same Quantile® scale, allowing teachers to determine which mathematical skills and concepts a student is ready to learn and those that will require additional instruction. Educators use this information to match students with learning resources at the right developmental level.

MetaMetrics built the Quantile scale and its taxonomy of skills and concepts in the hope of advancing how mathematics instruction is targeted to meet the needs of individual learners. As illustrated in Figure 1, today's classrooms comprise wide ranges of student reading and mathematical abilities. For example, 50 percent of fourth graders have a Quantile measure between 495Q and 815Q. Said differently, there is a range of 320Q within the fourth grade; 25 percent of students are above 815Q and 25 percent are below 495Q (this is comparable to an interquartile Lexile range of 365L). By manipulating certain elements of instruction, educators can support the needs of all learners, regardless of where they are in their mathematical thinking and development.

Mathematics, by its nature, builds upon a set of initial skills and concepts. Within the Quantile Framework, MetaMetrics has identified and empirically ordered approximately 500 mathematical skills and concepts called *QTaxons*. The QTaxons extend over the five content strands proposed by the National Council of Teachers of Mathematics (NCTM)—geometry, measurement, numbers and operations, algebra/ patterns & functions, and data analysis & probability—and range from kindergarten skills through Algebra II topics. Each QTaxon has a unique *knowledge cluster* (Figure 2) that illustrates different types of relationships and the connections across strands that create a cohesive mathematical understanding. Knowledge clusters include precursory skills (prerequisite knowledge), supporting (or supplementary) skills, and impending skills (what follows a particular skill). For example, in order to add two- and three-digit numbers without regrouping (QT79), a student first should understand addition and subtraction facts to ten

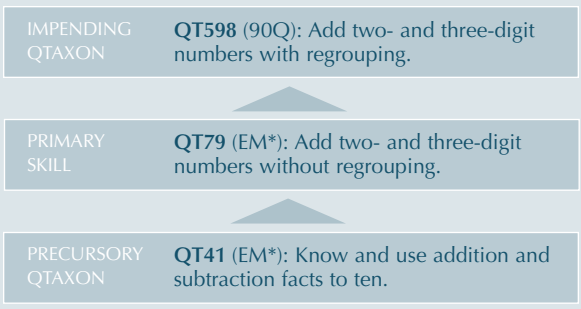
Figure 1: Typical Grade Ranges for Students' Quantile and Lexile Measures

Grade	Quantile Measures (Interquartile[1] Range, Mid-Year)	Lexile Measures (Interquartile[1] Range, Mid-Year)
K–1	EM* (0Q and below)	Up to 300L
2	100Q to 480Q	140L to 500L
3	340Q to 660Q	330L to 700L
4	495Q to 815Q	445L to 810L
5	635Q to 955Q	565L to 910L
6	700Q to 1020Q	665L to 1000L
7	750Q to 1070Q	735L to 1065L
8	820Q to 1140Q	805L to 1100L
9 (Algebra)	870Q to 1190Q	855L to 1165L
10 (Geometry)	940Q to 1260Q	905L to 1195L
11 (Algebra II)	1000Q to 1320Q	940L to 1210L**

*Emerging Mathematician **Grades 11 and 12

[1]Interquartile is the range between the upper and lower quartiles (i.e., the middle 50 percent of a distribution) and is equal to the difference between the 75th and 25th percentile.

Figure 2: Knowledge Cluster



*Emerging Mathematician (EM) represents a Quantile measure of 0Q and below.

Leveling the International Playing Field

The Quantile Framework for Mathematics can assist educators and students as they strive to develop what mathematics scholar Liping Ma (2007) describes as a "profound understanding of fundamental mathematics." It has been well documented that Chinese students typically outperform U.S. students on international comparisons of mathematics competency, despite the fact that Chinese teachers receive far less education than their U.S. counterparts (11–12 years versus 16–18 years). Ma believes that elementary teachers need to become accomplished mathematics teachers and suggests that teaching knowledge is more common abroad because Chinese educators begin their careers with a better understanding of elementary mathematics and, more importantly, continue to improve how they teach mathematics throughout their careers. She notes that teaching conditions in the U.S., unlike those in China, have a substantial effect on the development of elementary teachers' mathematical knowledge. Ma recommends that changes in teacher preparation, support and mathematics education research might allow U.S. teachers to attain this "profound understanding of fundamental mathematics."

In his book, "Outliers: The Story of Success (2008)," Malcolm Gladwell notes that, on average, Asian eighth-grade students work on mathematical problems much longer than U.S. students before they deem the problem too difficult to solve. Gladwell cites this persistence factor as the major difference in test scores across countries and reports that this is even reflected in the questionnaire that students complete prior to taking the Trends in International Mathematics and Science Study (TIMSS) test. TIMSS compares achievement in mathematics and science of U.S. students in grades four and eight with that of students in other countries. The rank order by questionnaire completion rate is exactly the same as the rank order of mathematics scores.

(precursory QTaxon (QT41)). Once these skills are learned, the student then is ready to add two- and three-digit numbers with regrouping (impending QTaxon (QT598)). Many students learn procedural knowledge without conceptual understanding. Precursory and supporting QTaxons supplement a lesson to deepen students' understanding of mathematical concepts.

QTaxons also are aligned with state standards. The Quantile Web site at www.Quantiles.com offers a number of free resources that support differentiated mathematics instruction (by state-defined accountability requirements), including more than 200 textbooks and 350 curriculum resources such as websites, games and books. These resources have been calibrated to the Quantile scale and have a Quantile measure. Educators use these Quantile measures to find mathematically appropriate material based on a specific topic and where a student is in his or her mathematical development. They also can use the online lesson planner to organize their lesson planning and include supplemental materials.

The Quantile Framework helps to validate what educators have known for some time: the range of mathematical abilities within any given classroom is significant and differentiation is necessary. Just as "one size does not fit all" in reading instruction, "one size does not fit all" when teaching mathematics. The Quantile Framework captures the interconnectivity of mathematics. With this cohesive understanding of how mathematical skills and concepts fit together, educators can differentiate classroom instruction to support successful learning experiences for all students.

For more information, a list of the assessments and programs that report Quantile measures, and curriculum resources and textbooks that have Quantile measures, visit www.Quantiles.com.

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MetaMetrics, Inc., a privately held educational measurement company, develops scientifically based measures of student achievement that link assessment with instruction, foster better educational practices, and improve learning by matching students with materials that meet and challenge their abilities. The company's team of psychometricians developed the widely adopted Lexile Framework for Reading; El Sistema Lexile para Leer, the Spanish-language version of the Lexile Framework; The Quantile Framework for Mathematics; and The Lexile Framework for Writing. In addition to licensing Lexile and Quantile measures to state departments of education, testing and instructional companies, and publishers, MetaMetrics delivers professional development, resource measurement and customized consulting services.

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